Resonance and Stellarator Design

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Outline

Motivation

ORBIT simulations

Resonance

Islands with no mode present

Alfven Modes

Chaotic loss due to Alfvén Modes

Conclusion

Main Results

• Resonances of high energy particles in magnetic confinement devices can strongly modify the particle distribution.
Resonance location depends on helicity of the particle orbit, which is energy dependent due to drifts.

• If a resonance matches the period of the equilibrium
Islands appear in particle orbits, increasing in size with energy

• A resonance always provides a site for an Alfvén mode.
Mode is likely unstable if there exist local high energy gradients.

• If there is toroidal dependence of B or $\partial B/\partial \psi$ an Alfvén mode produces chaotic orbits of particles independent of energy or pitch.

Hamiltonian Guiding Center Code

The covariant expression for the magnetic field used in the guiding center code Orbit is

$$\vec{B} = g(\psi)\nabla\zeta + I(\psi)\nabla\theta + \delta(\psi,\theta,\zeta)\nabla\psi,$$

θ and ζ are poloidal and toroidal coordinates and ψ is the toroidal flux.
$g(\psi)$ is the total poloidal current outside the surface ψ,
including the current in the field coils.
$I(\psi)$ is the total toroidal plasma current inside ψ.
δ is a result of non orthogonal coordinates.

The Hamiltonian is $H = \rho_{\|}^2 B^2/2 + \mu B + \Phi$
with $\rho_{\|} = v_{\|}/B$. Alfvén mode $\delta\vec{B} = \nabla \times \alpha\vec{B}$
Lagrangian with an Alfvén mode present

$$L = (\psi + \rho_{\|} I + \alpha I)\dot{\theta} + (\rho_{\|} g + \alpha g - \psi_p)\dot{\zeta} + \mu\dot{\xi} - H(\psi_p, \rho_{\|}, \theta, \zeta),$$

Resonance

For resonance a particle orbit must periodically return to experience the same perturbation.
Thus, at some time T

$$\theta(T) - \theta(0) = 2\pi n, \ \zeta(T) - \zeta(0) = 2\pi m, \ \psi(T) = \psi(0),$$

for integers m, n. The frequency is $\omega = 2\pi/T$,
Launch passing particles distributed in E, μ, ψ, ζ, $\theta = 0, \pi$
Poloidal transits $n = mod(\theta(t) - \theta(0), 2\pi)$
and toroidal transits $m = mod(\zeta(t) - \zeta(0), 2\pi)$ when orbit returns
Helicity m/n is equal to field line helicity q at low energy
Drift causes resonance to move in minor radius with energy change
Resonance can enter or leave the plasma

3/2 Resonance

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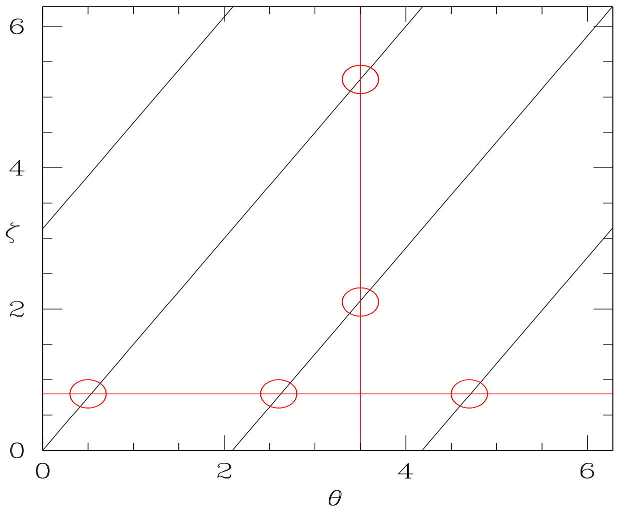


Trajectory of a 3/2 resonant particle through multiple transits, starting at 0,0 .
Three elliptic points found in Poincaré section in θ, fixed ζ.
Two elliptic points found in Poincaré section in ζ, fixed θ.
Normally it is the ζ dependence of a stellarator equilibrium that can match the resonance.

Particle Drift

The particle motion in ψ is only drift motion, second order in $\rho_\parallel$

$$\dot{\psi} = -\frac{g}{D}(\mu + \rho_\parallel^2 B)\frac{\partial B}{\partial \theta}.$$

The particle motion in the poloidal and toroidal directions are

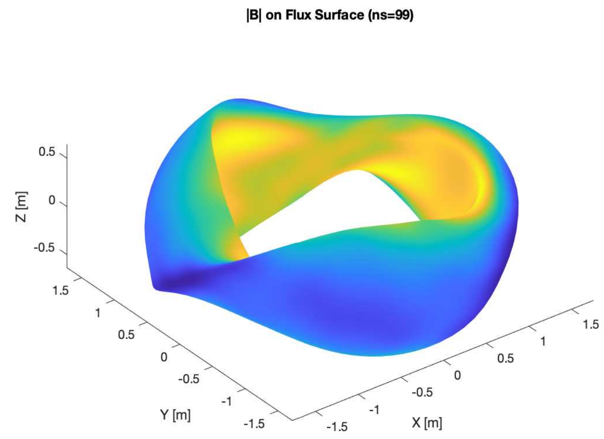
$$\dot{\theta} = \frac{\rho_\parallel B^2}{D}(1 - \rho_\parallel g') + \frac{g}{D}(\mu + \rho_\parallel^2 B)\frac{\partial B}{\partial \psi_p}$$

$$\dot{\zeta} = \frac{\rho_\parallel B^2}{D}(q + \rho_\parallel I'_{\psi_p}) - \frac{I}{D}(\mu + \rho_\parallel^2 B)\frac{\partial B}{\partial \psi_p}.$$

where $D = gq + I + \rho_\parallel(gI'_{\psi_p} - Ig'_{\psi_p})$.

At low energy (first order in $\rho_\parallel$) orbit helicity $\dot{\zeta}/\dot{\theta} = q(\psi)$
As the particle energy increases orbit helicity
can either increase or decrease due to drift depending on the orbit.

NCSX



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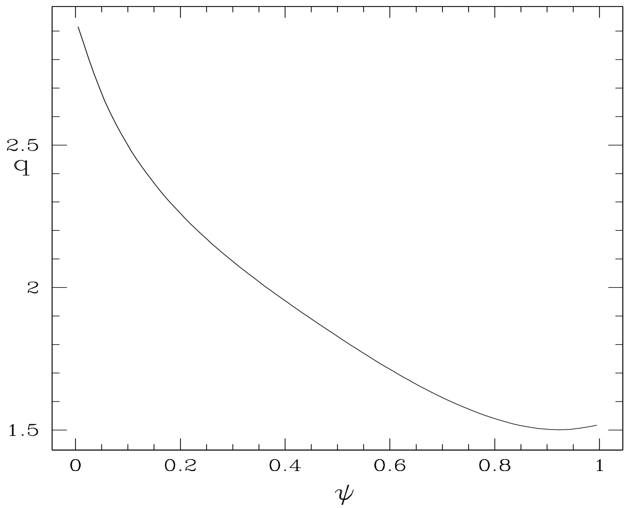


NCSX.

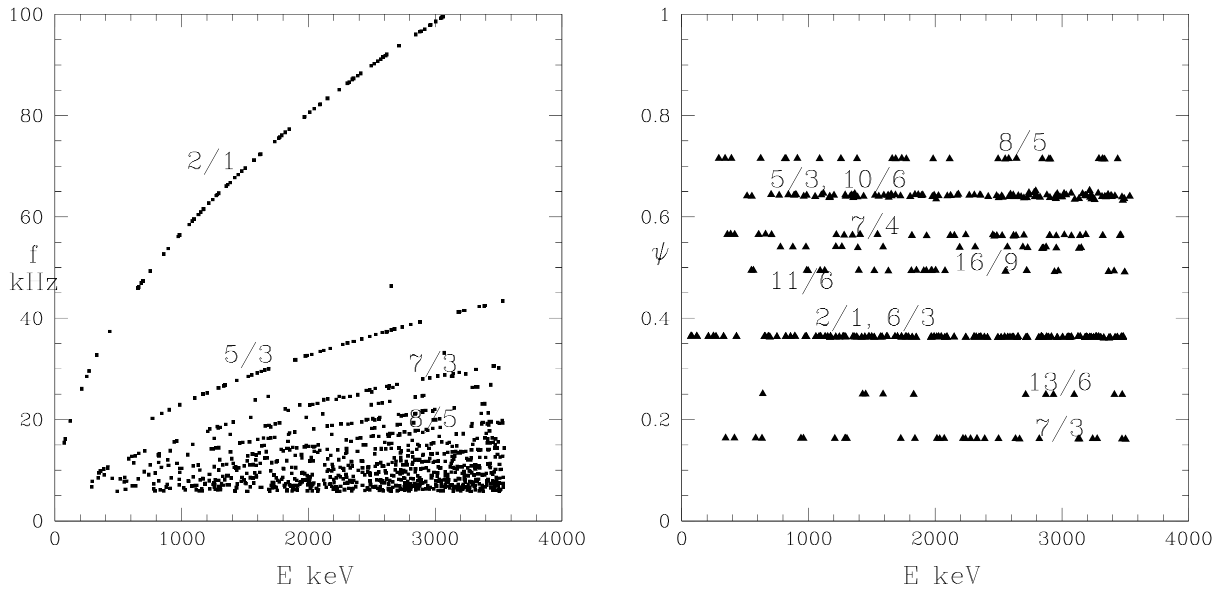
The equilibrium for NCSX has a major radius of 145 cm and magnetic field of 15 kG.

The field has a toroidal period of 3

The q profile has a maximum of 3. near axis and falls to 1.5 at the edge.

This device was designed but not built.

Energy dependance of NCSX negative frequency resonances



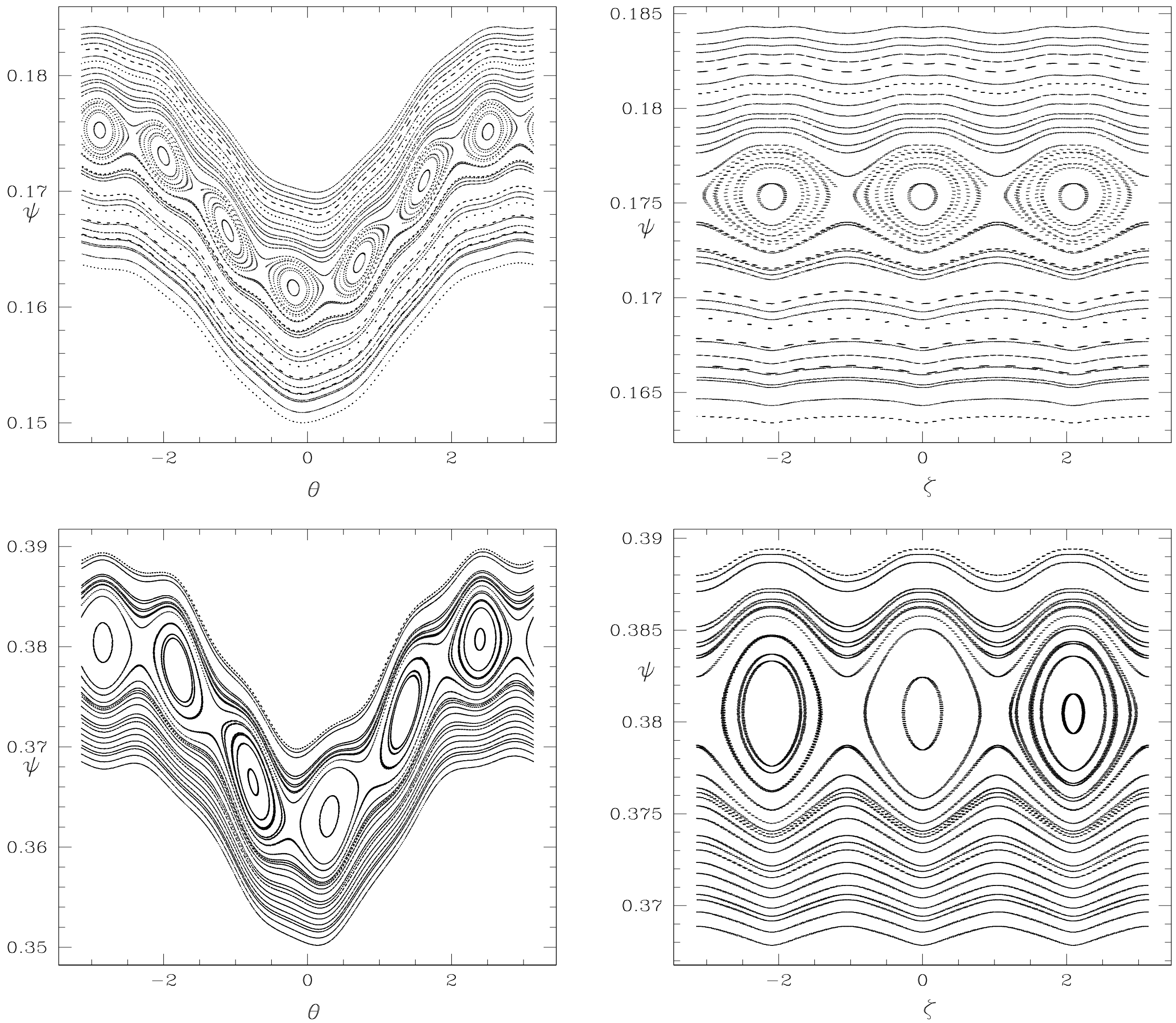
Negative frequency resonances in NCSXR vs particle energy

Matching B are the 5/3, 6/3, 7/3, 8/3 and higher helicity resonances

At low energy there are also 2/1, 5/2, 7/4, 8/5

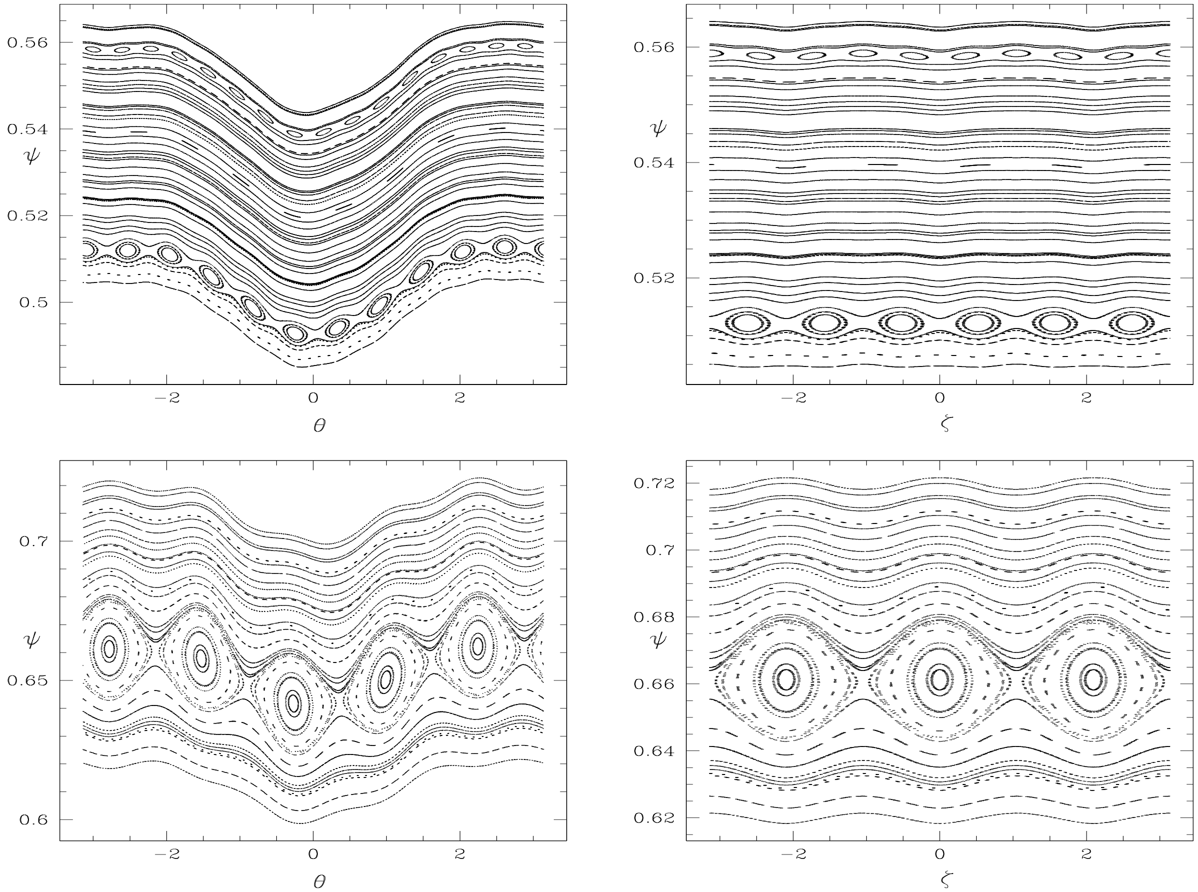
and other resonances present, not matching B.

Resonance frequency scales as $1/l$ and the square root of the energy.

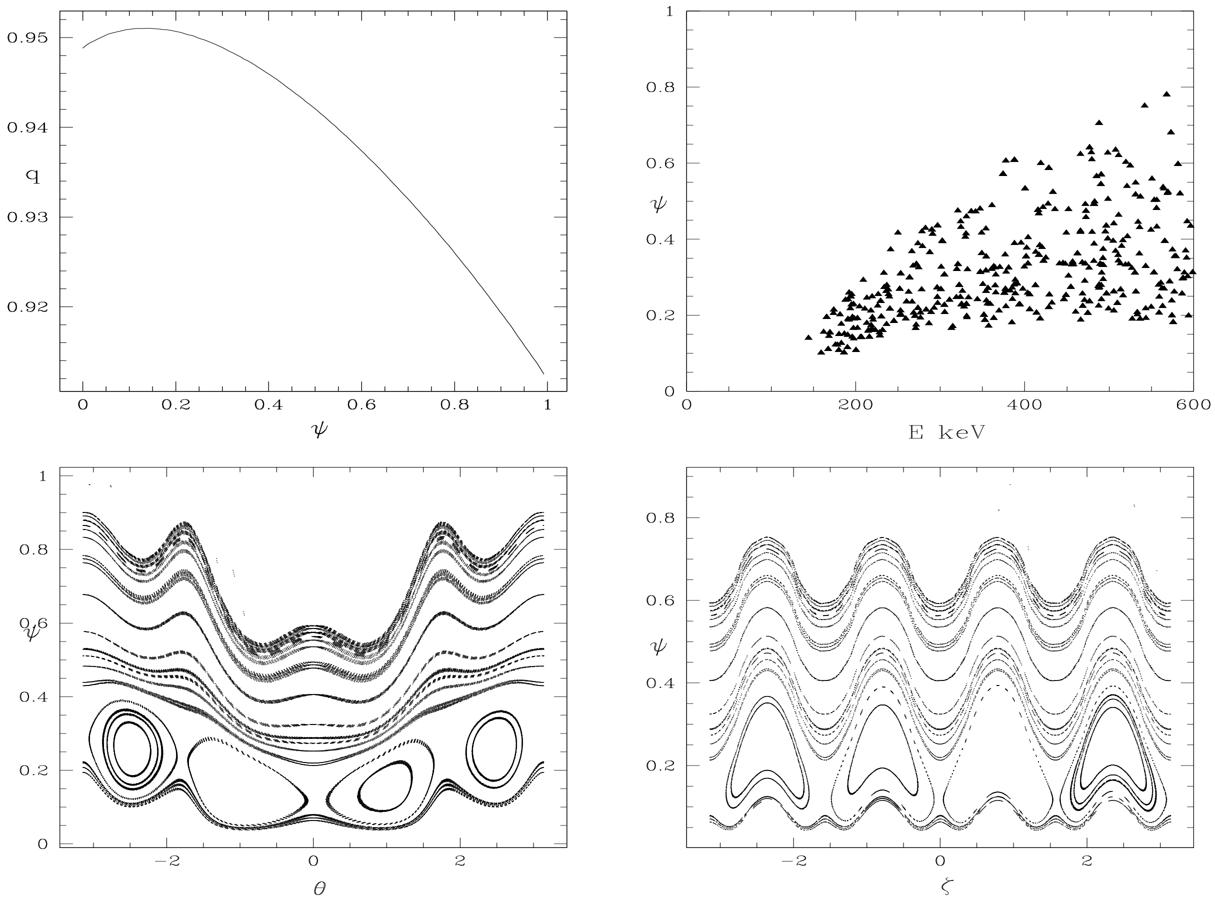


NCSXR Poincaré sections of unperturbed counter moving orbits at 3.5 MeV showing 7/3 and 6/3 resonances.

Nonzero width with no eigenmode. Resonance with toroidal field



NCSXR Poincaré sections of deeply counter passing orbits at 3.5 MeV showing 16/9, 11/6 and 5/3 resonances.
Islands twice this size for barely passing- Thomas
Large radial field moves resonance, no change in size.



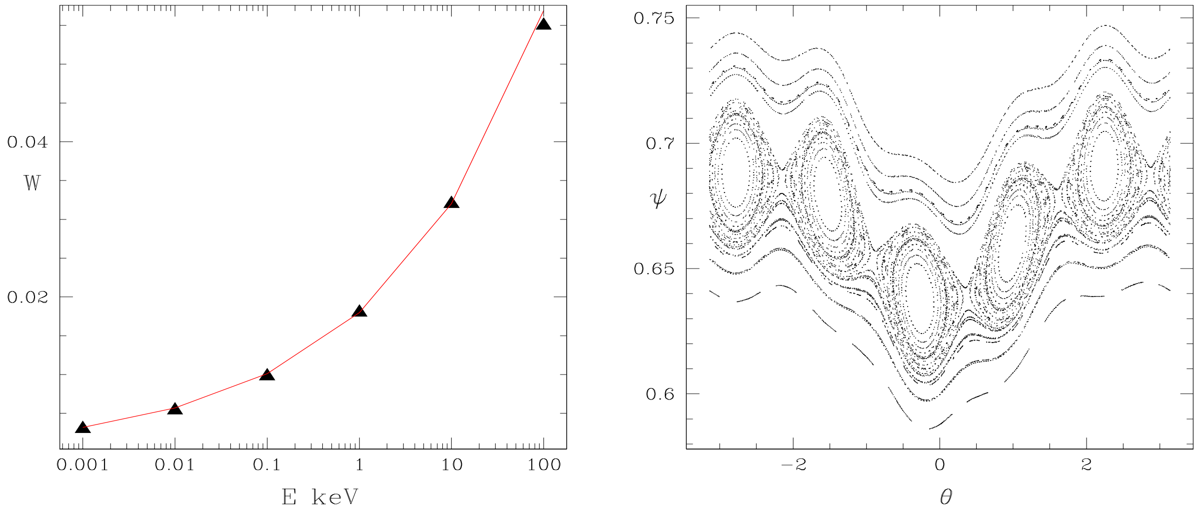
HSX stellarator, Wisconsin B= 10 kG, R = 165 cm

q profile just below q=1. Equilibrium has toroidal period 4

The 4/4 resonance with helicity 1 jumps into plasma from axis at about 150 keV and moves out as energy increases

Poincaré sections of 200 keV deeply counter passing orbits

Adiabatic invariant $J(\alpha)$ Parra Diaz and Thomas



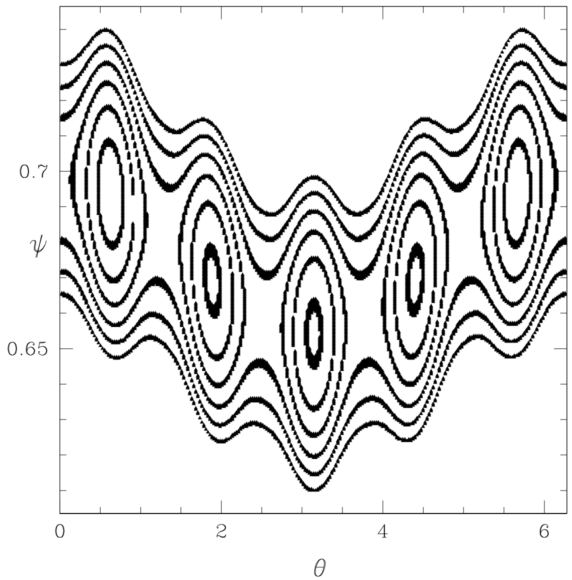
NCSX width vs particle energy $W = .018E^{1/4}$ and Poincare 100 keV. There is an adiabatic invariant $J(\alpha)$ given by an integral over the length of the low energy resonance trajectory,

$$J(\alpha) = \int_0^L dl \frac{\sqrt{2E - 2\mu B}(gq + I)}{Bq} - \pi n q'(\psi_r) \frac{(\psi - \psi_r)^2}{q(\psi_r)}$$

with $\theta = \alpha + \zeta/q(\psi_r)$, $B = B(\psi_r, \theta, \zeta)$ and ψ_r the resonance flux location. The parameter α thus gives surfaces away from the resonance field line.

Shift of surface with energy given by $J(\alpha)$ by higher order terms

But can be simply obtained–
$\dot{\zeta}$ is unchanged by energy increase
but $\dot{\theta} \simeq \dot{\theta}_0 + 2E\partial_\psi B/Bq$,
changes orbit helicity by



$$h = q(\psi_r)(1 - \frac{\sqrt{2E}}{B}\partial_\psi B).$$

This must be balanced by a change in ψ, because the orbit must close on itself.
A shift in position changes helicity through $q(\psi) = q(\psi_r) + q'(\psi_r)(\psi - \psi_r)$, and this must balance the change due to energy giving

$$\psi = \psi_r + \frac{q\sqrt{2E}}{q'B}\partial_\psi B$$

$cos(\theta)$ shift from $\partial_\psi B$
of resonance shown for 100 keV in NCSX.

Orbital Chaos

Alfvén mode in a Stellarator without a symmetry
A Poincaré frame ψ, θ is at times $n\zeta - \omega t = 2\pi k$
For a coherent Poincaré plot the step must depend only on ψ, θ
Time step depends on the value of ζ through $B(\psi, \theta, \zeta)$

$$\dot{\psi} \simeq mgB(\psi, \theta, \zeta) v_{\|} \alpha(\psi) cos(\Omega), \qquad \dot{\rho}_{\|} \simeq v_{\|}^2 \partial_{\psi} B(\psi, \theta, \zeta) \alpha(\psi) sin(\Omega).$$

for fixed ψ, θ with $\Omega = n\zeta - m\theta - \omega t$.
ζ is given by ωt so there is a random step in ψ and in $\rho_{\|}$ given by

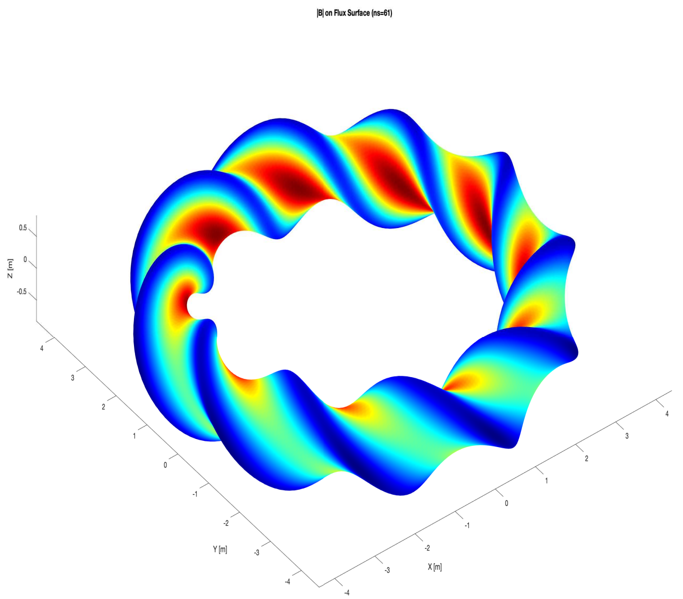
$$\dot{\psi} = m v_{\|} \delta[B] \alpha(\psi), \qquad \dot{\rho}_{\|} = v_{\|}^2 \delta[\partial_{\psi} B)] \alpha(\psi)$$

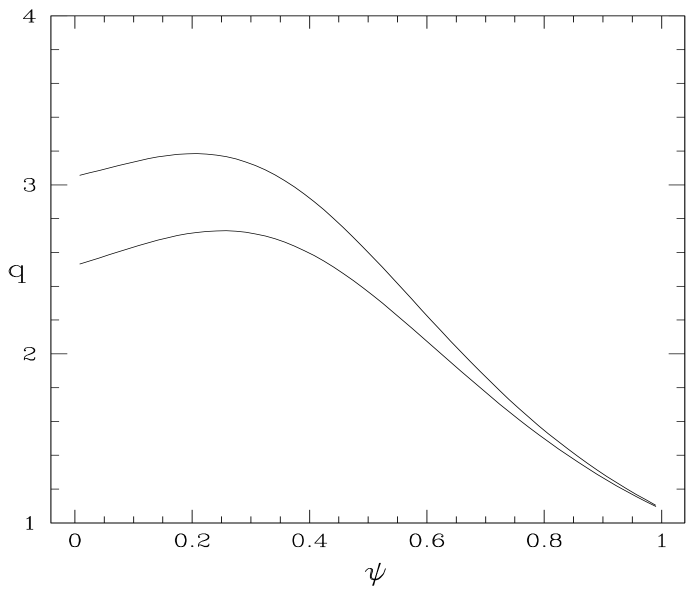
with $\delta[X]$ equal to X minus the mean value of X over ζ.
Diffusion in ψ and energy of all particles where the eigenfunction large.
The modulation of B is typically ten percent so this introduces ten percent random noise onto the pertubation.
Enough to destroy any resonance island structure.
Need to redo mode growth and saturation theory

Resonant Electromagnetic Modes

- High frequency Alfvén modes cause particle redistribution and loss.
- Modes can only be destabilized where there is a resonance.
- Mode growth and saturation depend on local gradients and damping.
- If a high frequency resonance exists there is high probability.
that a distribution of high energy particles will destabilize
a mode at this location.
- Resonances can occur for co-moving or counter-moving orbits.
They are very different because of drifts. Must examine both cases.
- In a symmetric device the distribution is modified in a
local coherent manner due to rotation about resonance elliptic points
- In a nonsymmetric device the distribution change is chaotic
with changes in growth and saturation.

LHD





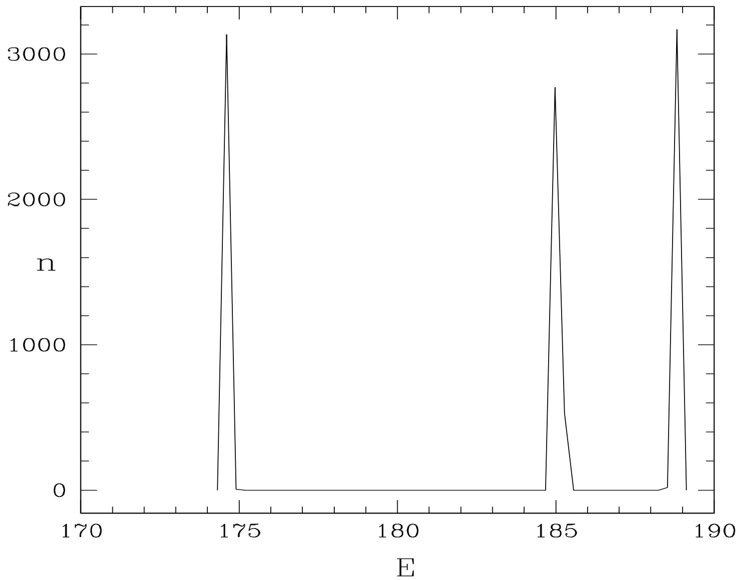
The Large Helical Device (LHD) is a fusion research device in Toki, Gifu, Japan

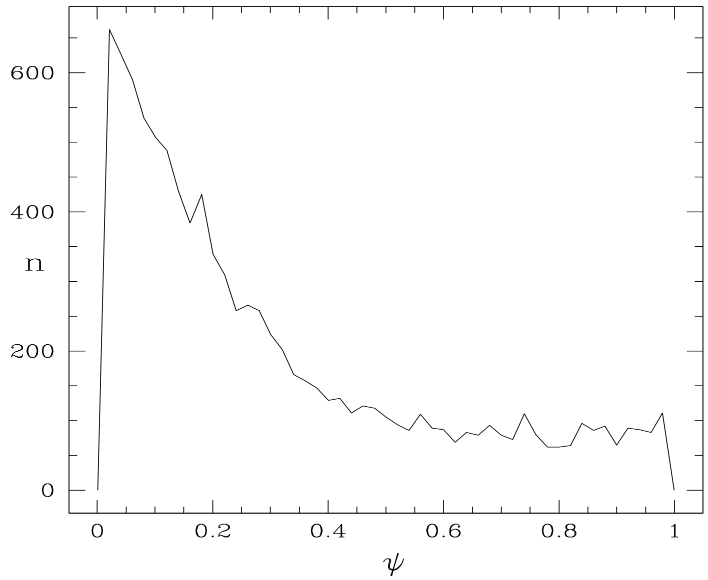
The LHD employs a heliotron field with toroidal period 10.

Typical q profiles during operation.

Examine Hydrogen beam discharge of 2010 and Deuterium beam discharge of 2022

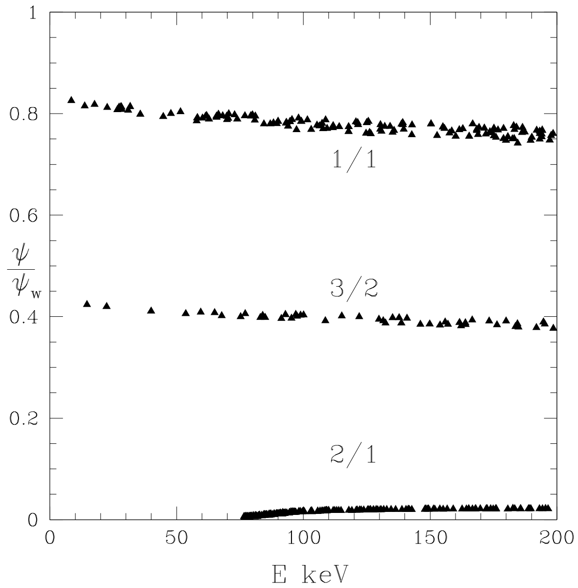
LHD 2010

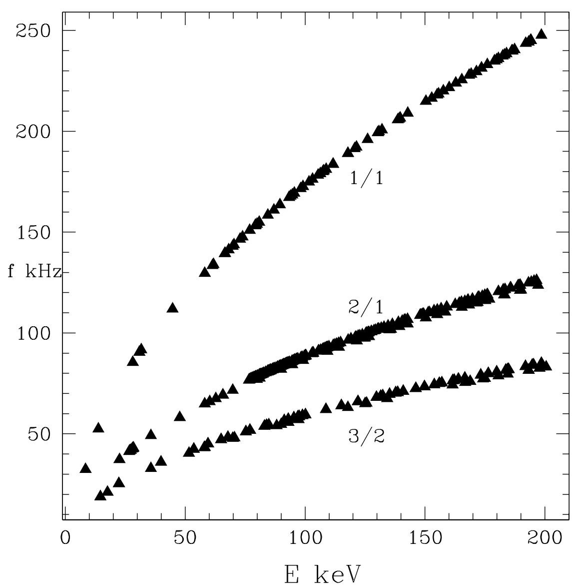




Energy and deposition location ψ of the particles
injected by the three Hydrogen beams
during the 2010 experiment.
Two beams were counter passing,
only the beam at 185 keV was co-passing.

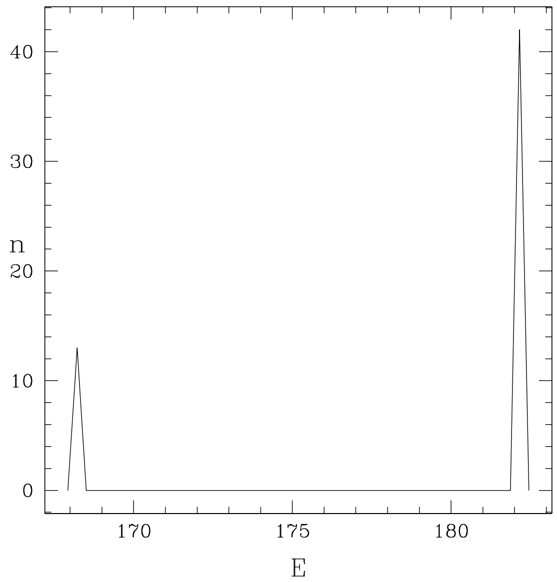
LHD 2010

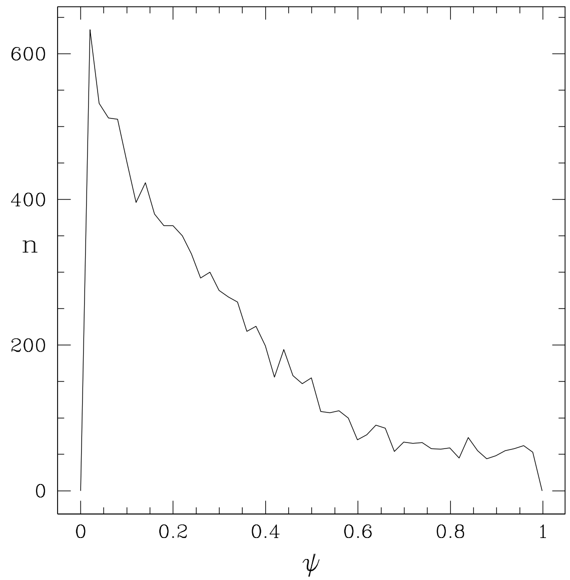




Resonances for the hydrogen beam discharge of 2010 showing location and frequency vs particle energy.
These resonances exist for deeply counter passing ions.
Frequency somewhat lower for smaller pitch.
Two of the three beams were counter and deeply passing.
Experimentally observed and numerically found (1/1) Alfvén instability at 70 kHz causing significant particle loss.

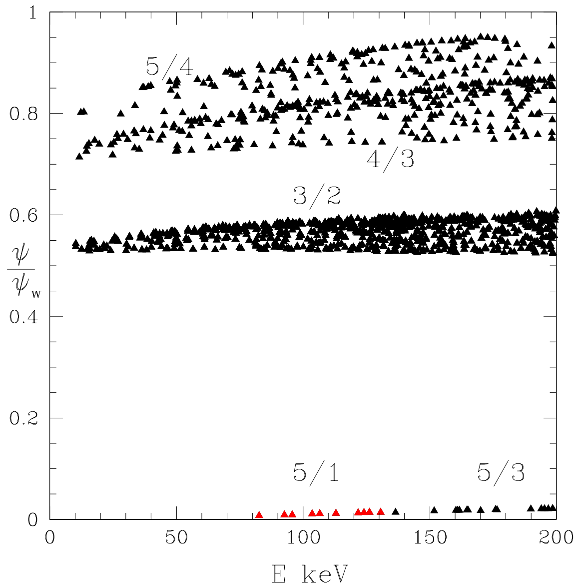
LHD 2022

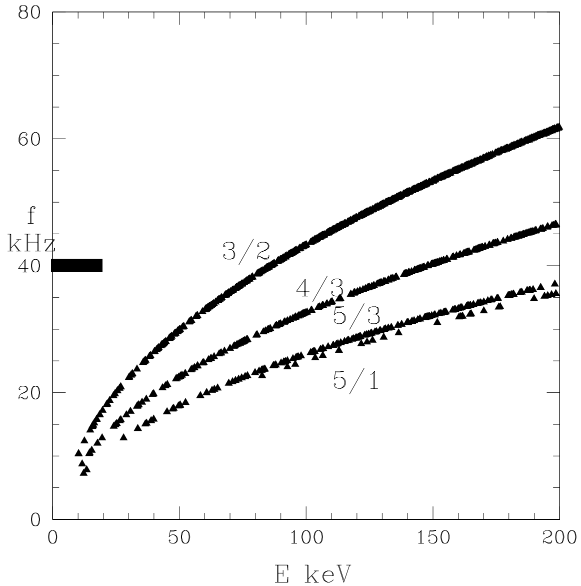




Energy and deposition location ψ of the Deuterium particles injected by the two beams during the 2022 experiment.
The high energy beam was deeply co-passing,
the low energy beam counter.

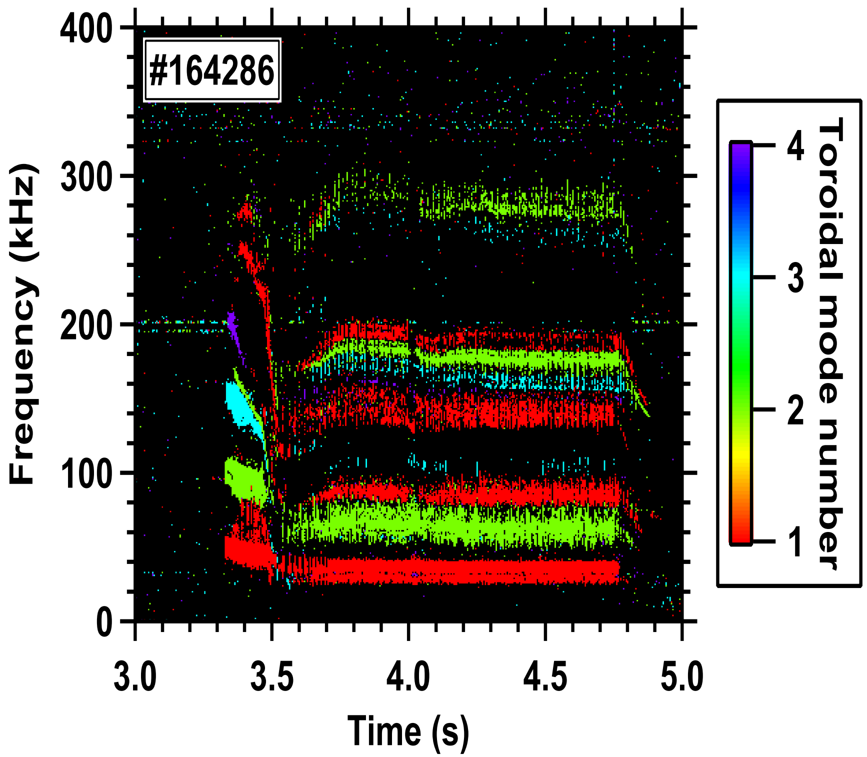
LHD 2022





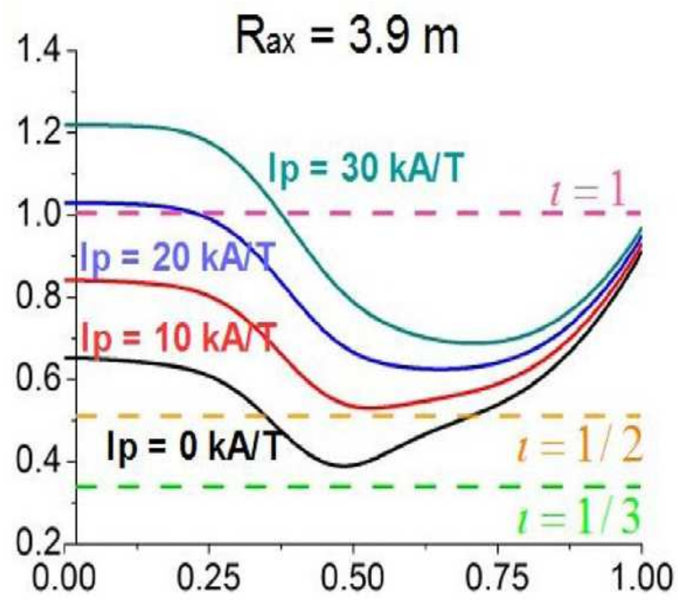
Resonances for the deuterium beam discharge of 2022
showing location and frequency vs particle energy.
These resonances exist for deeply co- passing ions.
Equilibrium toroidal period is ten
Experimentally observed 40 kHz avalanche with particle loss,
modes not identified but included $n = 1$ and probably also 3/2 and 4/3.
Conclude that the presence of a low mode number resonance and
high energy particles will very likely destabilize an Alfvén mode.

LHD observed modes 2022



Determination
of modes in LHD 2022
An avalanche
was observed
with significant
particle loss

LHD Alfvén mode free?



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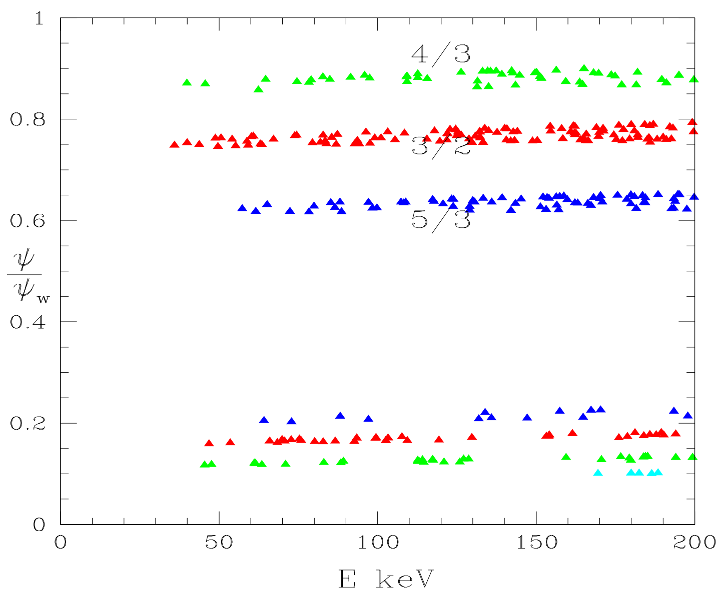


Helicity profile controlled by beams

Look for profile with no low order resonances

Use 10 kA profile

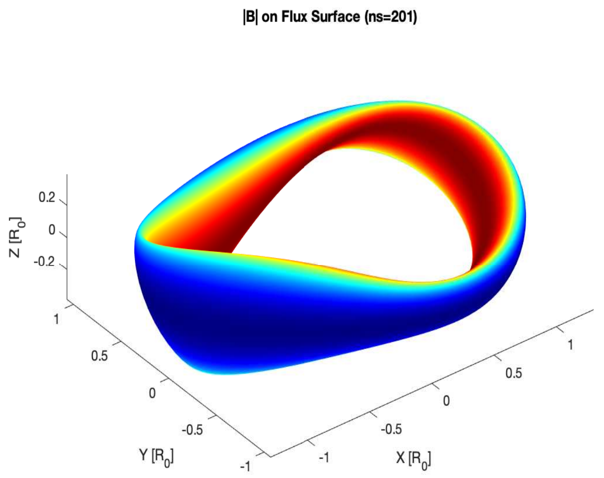
Can avoid q = 1 and q = 2

But strong resonances in outer plasma

5/3, 3/2, and 4/3

Avoiding Alfvén modes in LHD not possible

Quasi Axisymmetric Stellarator



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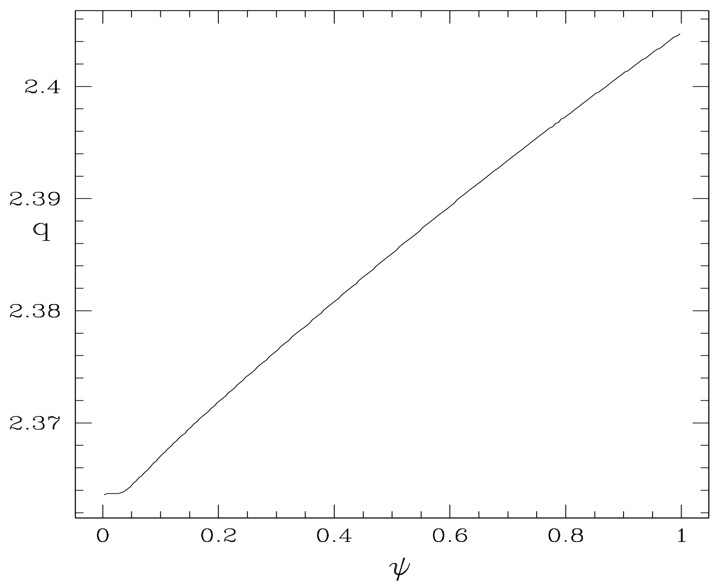
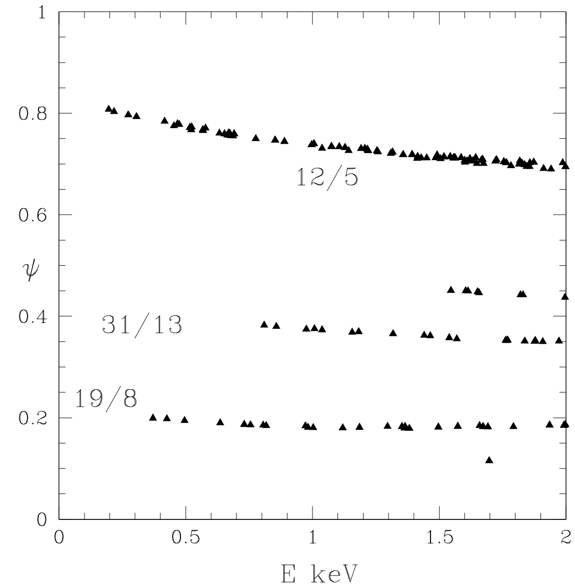


Quasi symmetric Stellarator design of Landremann and Paul Field helicity profile $q(\psi)$ with 12/5 the only small rational in the plasma.

Coherent Poincare in QA with small Alfvén mode



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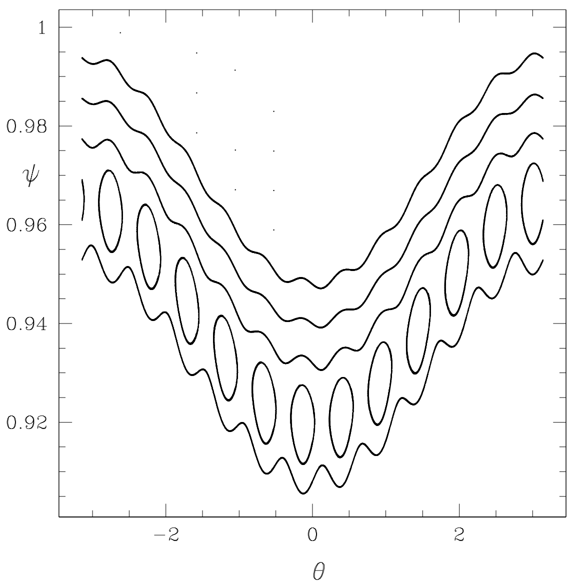
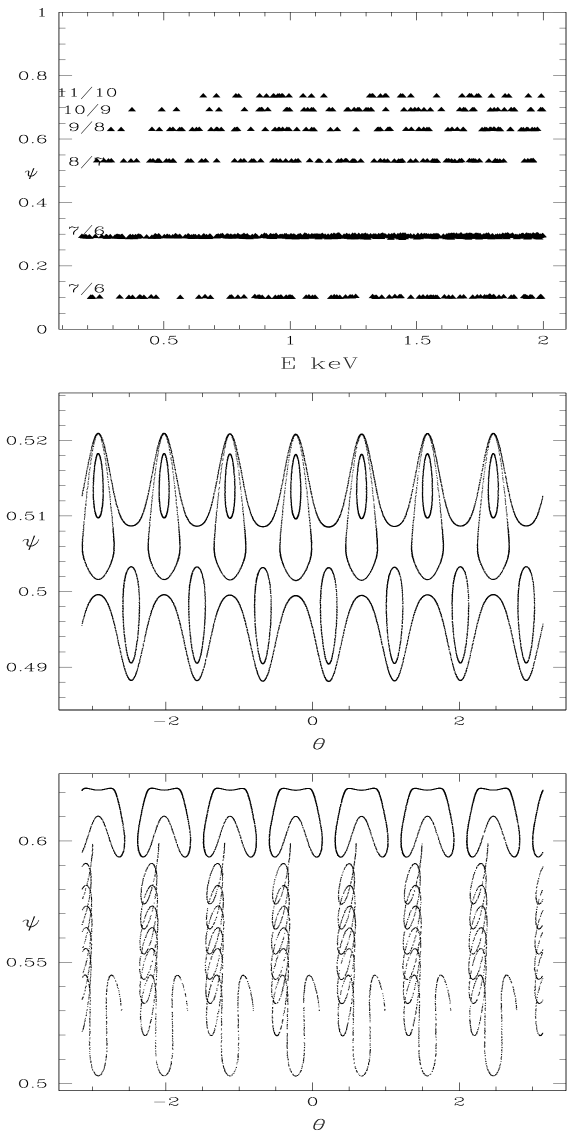


Small amplitude 12/5 Alfvén mode produces resonance islands for counter moving passing particles near plasma edge.
QA is sufficiently symmetric for a coherent Poincaré plot.
Amplitude $A = 3 \times 10^{-5}$,
No diffusion due to small Alfvén mode.

Chaotic orbits in W7X with small Alfvén mode



There is a strong 7/6 negative frequency resonance in W7X. Alfvén mode with structure of a localized Gaussian in ψ. Because of the toroidal dependence of B it is impossible to produce clear Poincare plots showing the resonance. A small amplitude 7/6 Alfvén mode showing isolated closed orbits, trajectory crossing and a form of accelerator motion near the resonance.

Conclusion

• Initially an equilibrium can be chosen so that there are no low order rationals in the field line helicity $q(\psi)$, so no high frequency resonances. This cannot be done in a tokamak. At high current q is near 1 on axis and increases outward, giving q of 3/2, 4/3, 2, 3 in the plasma.

•As particle energy increases resonances can emerge from the magnetic axis or the plasma edge, and move into the plasma.

•If the equilibrium has significant toroidal variation and resonances match the toroidal B, islands form even with no unstable mode, width increasing with particle energy.

• Alfvén modes are easily destabilized at resonances.

• Following LHD results we conjecture that Alfvén modes will often appear where there is a high frequency resonance.

•If the equilibrium is not symmetric a mode produces chaotic diffusion of all energies and pitch at small mode amplitude.

• Avoiding resonance islands must become a part of stellarator design. Symmetrization requirement is severe